Caliphate Journal of Science & Technology (CaJoST)

Open Access Journal available at: https://cajost.com.ng/index.php/files and https://www.ajol.info/index.php/cajost/index

DOI: https://dx.doi.org/10.4314/cajost.v6i3.3

Article Info

Received: 8th April, 2024

Revised: 30th October, 2024

Accepted: 7th November, 2024

Precision and Accuracy Determination of Horizontal Geodetic Networks Using Least Squares Technique

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Cite this: *CaJoST*, 2024, 3, 265-272

The determination of precision and accuracy in horizontal geodetic networks using a least squares adjustment is a complex process that has a range of available software applications. However, it is commonly observed that users may not possess a complete understanding of the underlying theories and equations that inform the development of such software. This paper presents a set of step-by-step procedures for determining the precision and accuracy of horizontal geodetic networks using the observation equation method of the least squares technique. In the sample calculation, the a posteriori variance and standard error were computed to obtain the precision and accuracy of the network, respectively. The inverse of the normal matrix was multiplied with the a posteriori variance to obtain the precision of the adjusted parameter. The square root of the precision was evaluated to obtain the accuracy of the adjusted parameter. The enumerated procedures are presented in a sequential order to ensure ease of comprehension for users. Additionally, a numerical example is provided to illustrate the procedures and to enhance understanding of the concepts presented. The computed adjusted network's precision and accuracy are 0.723m^2 and 0.850m respectively. Our objective is to provide users with the necessary tools and knowledge required to confidently perform this task.

Keywords: precision, adjustment, geodetic network, accuracy, least squares

1. Introduction

The Least Squares technique is an essential tool for analyzing and correcting random errors in survey measurements. According to Zulkifli *et al*. [1], the Least Squares Adjustment (LSA) method is an advanced adjustment technique that refines observations based on the principles of probability, which account for the occurrence of random errors. By utilizing estimated precisions of observation coordinates, LSA calculates adjusted positions that reconcile discrepancies between the observed and adjusted coordinates.

Furthermore, this method produces comprehensive statistical data on the adjustments made, enabling an assessment of the strength and confidence in computed positions, while also facilitating the detection of potential blunders. As a rigorous approach, it is particularly valuable in the adjustment of horizontal geodetic networks, yielding the most accurate and reliable values for survey measurements. The Least Squares technique guarantees the determination of the most probable values of observations based on a given set of redundant measurements [2].

A horizontal geodetic network consists of a series of reference stations, each of which has its position and coordinates established with a high degree of accuracy. The network is categorized into three groups: first-order, second-order, and third-order. The classification is based on the precision with which the networks are established. The accuracy and precision of the network are typically calculated using the least squares adjustment method.

Researchers have encountered challenges in comprehending the accuracy and precision of the horizontal geodetic network when employing the least squares technique. However, fortunately, there are software programs available to carry out the adjustment. Despite this, many users still struggle to grasp the theory and equations employed in developing these programs. The matrix nature of this technique has made it particularly challenging. The least squares

adjustment can be represented as the sum of the estimate matrix and the observation matrix, equating to the residual matrix. Unfortunately, obtaining each of these matrices has posed a significant problem. Previous studies [3,4,5,6] that have employed the least squares adjustment technique have failed to provide a breakdown of how this technique is applied. To assess the accuracy and precision of the adjusted network, it is necessary to obtain the residual matrix and have redundant observations, as well as a degree of freedom.

This paper describes the observation equation method of the least squares adjustment technique for determining the precision and accuracy of horizontal geodetic networks established using DGPS. The application is presented in a detailed, step-by-step manner.

1.1 Observation Equation Technique of Least Squares Adjustment

The observation equations are mathematical expressions that link observed quantities to independent unknown parameters and observational residuals. Each observation has a unique set of unknowns and requires a corresponding equation. To arrive at a unique solution for these unknowns, the number of equations must equal the number of unknowns. In most cases, there are more observations than unknowns, allowing the principle of least squares to determine the most probable values for the unknowns [7].

Ayeni [8] and Okwuashi and Asuquo [9] have described the observation equation method, which expresses adjusted observations as a function of adjusted parameters. As outlined in [10], the functional relationship between adjusted observations and adjusted parameters is established through this approach (Equation, 1).

$$L_a = F(X_a) \tag{1}$$

Where,
L_a = adjusted observations
X_a = adjusted parameters

Equation (1) is a linear function and we have derived the general observation equation model. To create a matrix expression for conducting a least squares adjustment, comparisons will be draw with systematic procedures. The matrix notation for the observation equation system can be found in [11] and [12].

$$V = AX - L \tag{2}$$

Where,
A = Design Matrix
X = Vector of Unknowns
L = Calculated Values (l_o) Minus Observed Values (l_b)
V = Residual Matrix.

That is,

$$V = \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix}, \qquad A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix},$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} l_1 \\ l_2 \\ \dots \\ l_m \end{pmatrix}$$

The process of determining the unknown parameters matrix, *X* involves the deduction of the normal matrix, *N* and the matrix of numeric terms, *t*. When dealing with weighted observations, the weight matrix, which is always a square matrix, also needs to be derived.

As stated by Ghilani [13], a system of weighted linear observation equations can be presented in matrix form as:

$$A^T WAX = A^T WL \tag{3}$$

To make *X* the subject of the formula, both sides of equation (3) will be divided by $A^T WA$. Thus,

$$X = (A^T WA)^{-1} A^T WL \tag{4}$$

If $A^T WA = N$, normal matrix and $A^T WL = t$, matrix of numeric terms, then equation (4) becomes

$$X = N^{-1} t \tag{5}$$

1.2 Weight of Uncorrelated Observation

Deakin [14] posits that the weight, *w* of an uncorrelated observation is directly proportional to the inverse of variance, σ^2. Hence,

$$w = \frac{1}{\sigma^2} \tag{6}$$

The weight matrix of uncorrelated observations is a diagonal matrix such that the off-diagonal elements are all zero and it is presented as

$$w = \begin{pmatrix} \frac{1}{\sigma_{11}^2} & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma_{12}^2} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_{nn}^2} \end{pmatrix}$$

1.3 Precision and Accuracy Determination

When calculating the mean precision and accuracy of a set of adjusted observations, a posteriori variance and standard error are used. It's worth noting that variance and standard error are measures of precision and accuracy, respectively. The formulae for calculating a posteriori variance, as outlined in [15] (Equation, 8), are utilized in this process.

$$\hat{\sigma}_o^2 = \frac{V^T WV}{r} \tag{8}$$

The model for the a posteriori standard error given in [16] and [12] is

$$\hat{\sigma}_o = \sqrt{\frac{V^T WV}{r}} \tag{9}$$

Where,
$r = n - m$ = degrees of freedom
n = number of observations
m = number of unknown parameters.

The precision and accuracy of the adjusted parameter and coordinates are derived from the variance-covariance matrix. The precision of the adjusted parameters is represented by the variances and elements of the principal diagonal, whereas the off-diagonal elements indicate the co-variances between specific coordinates. The square root of the variances determines the accuracy of the adjusted parameters. To calculate the variance-covariance matrix, the inverse of the normal matrix is multiplied by the a posteriori variance.

$$\sum_{XX} = \hat{\sigma}_o^2 N^{-1} \tag{10}$$

The trace is a commonly used metric for assessing accuracy in statistical models. Specifically, it refers to the sum of the diagonal elements of the variance-covariance matrix.

$\sum_{XX}$. It is usually written as $\mathrm{tr}\sum_{XX}$ or sometimes $\mathrm{tr}(\sum_{XX})$, that is,

$$\mathrm{tr}(\sum_{XX}) = \sum \hat{\sigma}_o^2 Q_{nn} \tag{11}$$

As stated in [17], one can interpret the trace of the variance-covariance matrix as a means of determining the accuracy of the related set of random variables. The calculations for determining the variance and standard error of the adjusted coordinates can be found in the models presented in [12].

$$\sigma_{xi}^2 = \hat{\sigma}_o^2 Q_{nn} \tag{12}$$

$$\sigma_{xi} = \sqrt{\hat{\sigma}_o^2 Q_{nn}} \tag{13}$$

Where,
Q_{nn} = inverse of the normal matrix main diagonal element.

Error ellipses are a common calculation in the adjustment of a horizontal or three-dimensional network. They provide a helpful measurement for interpreting the accuracy of positional station directions. Error ellipses are also commonly applied in optimizing a given network.
Mikhail and Gracie [18] gave the semi-major axis $\sigma_{x^1}^2$, semi-minor axis $\sigma_{y^1}^2$ and the orientation of the error ellipse θ as

$$\begin{aligned} \sigma_{x^1}^2 &= \frac{\sigma_x^2 + \sigma_y^2}{2} + \left[\frac{(\sigma_x^2 - \sigma_y^2)^2}{4}\right] + \sigma_{xy}^2 \\ \sigma_{y^1}^2 &= \frac{\sigma_x^2 + \sigma_y^2}{2} - \left[\frac{(\sigma_x^2 - \sigma_y^2)^2}{4}\right] + \sigma_{xy}^2 \end{aligned} \tag{14}$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \tag{15}$$

2. Methodology

2.1 Steps to Compute Precision and Accuracy

The following steps are necessary for the adjustment of a GPS network:

1. At least two fixed controls must be used to observe each network point. The data acquired with respect to any of the controls/base stations should be processed separately to obtain the coordinates of the new points.

2. After obtaining the coordinates of the new points, the observation equations must be deduced. The number of observation equations must not be less than the number of baselines.

3. From the deduced observation equations, the coefficient and design matrix (A), observation matrix (L), residual matrix (v), and matrix of unknown parameters (X) must be derived.

4. The weight matrix should also be deduced using Equation (6). The variances of the baseline vectors for DGNSS observation are obtained from the variance-covariance matrix of the processed DGNSS data.

5. After deducing the matrices, the unknown parameters must be computed using Equation (5).

6. The design matrix is used to multiply the computed values of the unknown parameters to determine the estimate as well as the most probable values.

7. The residuals must be evaluated by finding the differences between the estimates and the observations using Equation (2).

8. The precision of the adjusted observations can be determined using Equation (8) after computing the residuals. The degree of freedom must be calculated to obtain the precision of the adjusted observations.

9. The mean accuracy of the adjusted observations can be determined using Equation (9) by taking the square root of the determined precision.

10. The accuracy of the adjusted network can also be determined using Equation (11).

11. The precision and accuracy of the adjusted coordinates can be determined using Equations (12) and (13), respectively, after determining the precision and accuracy of the adjusted observations.

12. The semi-major axis, semi-minor axis, and orientation of the error ellipse can be computed using Equations (14) and (15), respectively.

3. Results and Discussion

3.1 A Sample Calculation of Precision and Accuracy

Table 1 shows the rectangular coordinates and corresponding variances for network stations A and B in relation to controls S and T, as determined through the use of DGPS. The positions for controls S and T remain 350472.960mE, 251374.548mN, and 354095.611mE, 251441.978mN correspondingly. Compute the most probable positions for points A and B, as well as the accuracy and precision of the observations and those of the adjusted positions using the least squares method. Additionally, compute the error ellipses for the coordinates of the stations.

Table 1: Observed Positions and their Respective Variances

Base Station	Rover Station	Coordinates (m)		Variance (m)	Base Station	Rover Station	Coordinates (m)		Variance (m)
S	A	**Northing**	250852.942	0.0000577	T	A	**Northing**	250852.949	0.0000465
		Easting	352598.178	0.0000314			**Easting**	352598.188	0.0000554
	B	**Northing**	252127.392	0.0000247		B	**Northing**	252127.398	0.0000338
		Easting	352572.216	0.0000822			**Easting**	352572.226	0.0000709

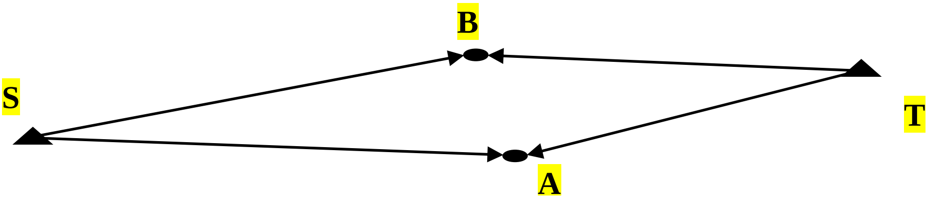

Figure 1: Observed Network Points

The method of least squares adjustment readily satisfies the requirement to accurately estimate three-dimensional coordinates from a redundant set of measurements.

Solution

Table 2: Determination of baseline vector

STATION FROM	ΔN (m)	ΔE (m)	COORDINATES		STATION TO
			NORTHING (m)	EASTING (m)	
			251374.548	350472.960	S
S	-521.606	2125.218	250852.942	352598.178	A
S	752.844	2099.256	252127.392	352572.216	B
			251441.978	354095.611	T
T	-589.029	-1497.423	250852.949	352598.188	A
T	685.420	-1523.385	252127.398	352572.226	B

Deduction of observation equations

$$A_N = S_N + \Delta N_{SA} + V_1$$
$$A_E = S_E + \Delta E_{SA} + V_2$$
$$B_N = S_N + \Delta N_{SB} + V_3$$
$$B_E = S_E + \Delta E_{SB} + V_4$$
$$A_N = T_N + \Delta N_{TA} + V_5$$
$$A_E = T_E + \Delta E_{TA} + V_6$$
$$B_N = T_N + \Delta N_{TB} + V_7$$
$$B_E = T_E + \Delta E_{TB} + V_8$$

Formulation of observation matrix, *L*, design matrix, *A,* residual matrix, *v*, weight matrix, *w* and matrix of unknown parameters, *X*,

$$A = \begin{pmatrix} A_N & 0 & 0 & 0 \\ 0 & A_E & 0 & 0 \\ 0 & 0 & B_N & 0 \\ 0 & 0 & 0 & B_E \\ A_N & 0 & 0 & 0 \\ 0 & A_E & 0 & 0 \\ 0 & 0 & B_N & 0 \\ 0 & 0 & 0 & B_E \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$L = \begin{pmatrix} S_N + \Delta N_{SA} \\ S_E + \Delta E_{SA} \\ S_N + \Delta N_{SB} \\ S_E + \Delta E_{SB} \\ T_N + \Delta N_{TA} \\ T_E + \Delta E_{TA} \\ T_N + \Delta N_{TB} \\ T_E + \Delta E_{TB} \end{pmatrix} = \begin{pmatrix} 250852.942 \\ 352598.178 \\ 252127.392 \\ 352572.216 \\ 250852.949 \\ 352598.188 \\ 252127.398 \\ 352572.226 \end{pmatrix}, \quad V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix},$$

$$W = \begin{pmatrix} 17331.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 31847.13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 40485.83 & & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12165.45 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 21505.38 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 18050.54 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 29585.80 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14104.37 \end{pmatrix}$$

$$, \quad X = \begin{pmatrix} A_N \\ A_E \\ B_N \\ B_E \end{pmatrix}$$

Computation of the unknown parameters, *X* using Equation (5)

$$\text{Normal matrix, } N = A^T WA = \begin{pmatrix} 38836.40 & 0 & 0 & 0 \\ 0 & 49897.68 & 0 & 0 \\ 0 & 0 & 70071.63 & 0 \\ 0 & 0 & 0 & 26269.82 \end{pmatrix}$$

,

$$N^{-1} = (A^T WA)^{-1} = \begin{pmatrix} 0.0000257 & 0 & 0 & 0 \\ 0 & 0.0000200 & 0 & 0 \\ 0 & 0 & 0.0000143 & 0 \\ 0 & 0 & 0 & 0.0000381 \end{pmatrix}$$

$$\text{Matrix of numeric terms, } t = A^T WL = \begin{pmatrix} 9742225063.763 \\ 17593829570.669 \\ 17666977195.008 \\ 9262009666.744 \end{pmatrix}$$

$$\therefore\ X = N^{-1}t = \begin{pmatrix} 0.0000257 & 0 & 0 & 0 \\ 0 & 0.0000200 & 0 & 0 \\ 0 & 0 & 0.0000143 & 0 \\ 0 & 0 & 0 & 0.0000381 \end{pmatrix} \times \begin{pmatrix} 9742225063.763 \\ 17593829570.669 \\ 17666977195.008 \\ 9262009666.744 \end{pmatrix} = \begin{pmatrix} 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \end{pmatrix}$$

Calculation of the most probable positions for stations A and B

$$AX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \end{pmatrix} = \begin{pmatrix} 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \\ 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \end{pmatrix}$$

Therefore, 250852.946mN, 352598.182mE and 252127.395mN, 352572.221mE are the adjusted as well as the most probable positions for points A and B respectively.

Evaluation of residuals, *v* using Equation (2)

$$V = AX - L = \begin{pmatrix} 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \\ 250852.946 \\ 352598.182 \\ 252127.395 \\ 352572.221 \end{pmatrix} - \begin{pmatrix} 250852.942 \\ 352598.178 \\ 252127.392 \\ 352572.216 \\ 250852.949 \\ 352598.188 \\ 252127.398 \\ 352572.226 \end{pmatrix} = \begin{pmatrix} 0.004 \\ 0.004 \\ 0.003 \\ 0.005 \\ -0.003 \\ -0.006 \\ -0.003 \\ -0.0005 \end{pmatrix}$$

The precision and accuracy of the adjusted network can be computed using Equations (8) and (9) respectively. As there are 4 unknowns and 8 observations, the degree of freedom, denoted by *r*, is equal to 4.

$$\hat{\sigma}_o^2 = \frac{2.891}{4} = 0.723\text{m}^2$$

$$\hat{\sigma}_o = \sqrt{\frac{2.891}{4}} = \sqrt{0.723} = 0.850\text{m}$$

The adjusted network's precision and accuracy are 0.723m^2 and 0.850m, respectively. The accuracy of the adjusted network was computed using Equation (11) which is the trace of the variance-covariance matrix.

$$\sum_{XX} = 0.723 \begin{pmatrix} 0.0000257 & 0 & 0 & 0 \\ 0 & 0.0000200 & 0 & 0 \\ 0 & 0 & 0.0000143 & 0 \\ 0 & 0 & 0 & 0.0000381 \end{pmatrix}$$

$$= \begin{pmatrix} 0.0000186 & 0 & 0 & 0 \\ 0 & 0.0000145 & 0 & 0 \\ 0 & 0 & 0.0000103 & 0 \\ 0 & 0 & 0 & 0.0000275 \end{pmatrix}$$

Thus, the accuracy of the adjusted network using Equation (10) is

$$\sum_{XX} = 0.0000186 + 0.0000145 + 0.0000103 + 0.0000275 = 0.0000709\text{m}^2$$
$$= \sqrt{0.0000709} = 0.00842\text{m}$$

Calculation of the precision and accuracy of the adjusted coordinates using Equations (12) and (13).

$$\text{Station A precision,} \begin{pmatrix} A_N \\ A_E \end{pmatrix} = 0.723 \begin{pmatrix} 0.0000257 \\ 0.0000200 \end{pmatrix} = \begin{pmatrix} 0.0000186 \\ 0.0000145 \end{pmatrix}$$

$$\text{Station B precision,} \begin{pmatrix} B_N \\ B_E \end{pmatrix} = 0.723 \begin{pmatrix} 0.0000143 \\ 0.0000381 \end{pmatrix} = \begin{pmatrix} 0.0000103 \\ 0.0000275 \end{pmatrix}$$

$$\text{Station A accuracy,} \begin{pmatrix} A_N \\ A_E \end{pmatrix} = \begin{pmatrix} \sqrt{0.0000186} \\ \sqrt{0.0000145} \end{pmatrix} = \begin{pmatrix} 0.00431 \\ 0.00381 \end{pmatrix}$$

Station *B* accuracy,

$$\begin{pmatrix} B_N \\ B_E \end{pmatrix} = \begin{pmatrix} \sqrt{0.0000103} \\ \sqrt{0.0000275} \end{pmatrix} = \begin{pmatrix} 0.00321 \\ 0.00524 \end{pmatrix}$$

Hence, the precision of northing and easting coordinates of stations *A* and *B* are respectively 0.0000186m^2, 0.0000145m^2 and 0.0000103m^2, 0.0000275m^2 while their respective accuracy are 0.00431m, 0.00381m and 0.00321m, 0.00524m.

Calculation of error ellipses of the station positions using equations (14) and (15) respectively.

For station *A*

$$\sigma^2_{x^1} = \frac{0.0000186 + 0.0000145}{2} + \left[\frac{(0.0000186 - 0.0000145)^2}{4}\right] + 0$$

$$= 0.000033\text{m}^2$$

$$\sigma^2_{y^1} = \frac{0.0000186 + 0.0000145}{2} - \left[\frac{(0.0000186 - 0.0000145)^2}{4}\right] + 0$$

$$= 0.000033\text{m}^2$$

$$\sigma_{x^1} = \sqrt{0.000033} = 0.00575\text{m}$$

$$\sigma_{y^1} = \sqrt{0.000033} = 0.00575\text{m}$$

$$\tan 2\theta = \frac{0}{0.0000186 - 0.0000145} \Rightarrow \theta = 0^o$$

The procedure was carried out again for Station B and the following values were obtained.

$$\sigma_{x^1} = 0.00615\text{m}$$

$$\sigma_{y^1} = 0.00615\text{m}$$

$$\theta = 0^o$$

Accordingly, the semi-major axis, semi-minor axis and the orientation of positions *A* and *B* are $0.00575\text{m},\ 0.00575\text{m},\ 0^o$ and $0.00615\text{m},\ 0.00615\text{m},\ 0^o$, respectively.

It should be noted that there is no covariance between stations A and B due to the uncorrelated nature of the observations. This results in zero orientation of the error ellipses.

4. Conclusion

The precision and accuracy of a horizontal geodetic network can be determined through the rigorous least squares method, which also provides information on the network's reliability and classification. This paper offers a step-by-step guide to applying the rigorous method and showcases a numeric example for a better understanding of the processing procedures. By following these procedures and examining the provided example, users can gain a comprehensive understanding of the least squares adjustment software and its associated theories and equations.

Conflict of interest

The authors declare no conflict of interest.

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